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DEVELOPMENT OF A P SYSTEM WITH ACTIVE MEMBRANES APPLYING MEMBRANE CREATION TECHNIQUES

SUNY Institute of Technology

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Summary

The P system is a general distributed model, highly parallel and based on the notion of a Membrane structure. This investigation addressed the development of a model for P systems with Active Membranes using the Membrane Creation technique. The usefulness of the model was shown by applying the model to solve the Hamiltonian Path Problem (HPP) for Undirected Graphs. The relevant algorithm for the problem is presented.

Introduction

P systems are a class of distributed parallel computing models introduced in (1), inspired by the way live cells process chemical compounds, energy, and information. In short, in the **regions** delimited by a **membrane structure**, one places **multi-sets of objects**, which evolve according to **evolution rules** associated with the regions. A **computation** consists of transitions among system configurations. The **result** of a halting computation is the vector of the multiplicities of objects present in the final configuration in a specified **output membrane** or of objects which leave the external membrane of the system (the **skin** membrane) during a computation.

The P systems can be of different forms, such as:

- 1) P system with Labeled Membranes
- 2) P system with Polarized Membranes
- 3) P systems with Active membranes.

In this investigation we addressed P systems with Active membrane defined in (2, 3). The P systems with Active Membranes can be classified in two forms:

- 1) Passive P systems, and
- 2) Active P systems.

The P systems addressed in (2, 3) are **Passive systems**. Here the membrane structure consists of m different membranes and during the computation this number may either decrease (by dissolving membranes) or increase (by dividing the existing membranes) but this number (with respect to the labels of membranes) is always less than or equal to m . The degree of a passive P system is the number of initial membranes present in the system.

Sometimes it may happen that different types of membranes may be increased in a membrane system by creating new membranes whose labels are different from the existing ones. Such systems are called **Active P systems**.

Membranes are created continuously in biology, for instance in the process of **Vesicle mediated transport**. Because one of the roles of membranes is to keep the molecules of a compartment close to each other in order to facilitate their reactions, when a compartment becomes too large, it often happens that new membranes appear inside it, more or less spontaneously or during biological evolution. The process of creating new

membranes is also known as “**Autopoiesis**” and the details of the process are addressed in (4, 5). This investigation considered the creation of membranes under the influence of existing objects. We abstracted the operation, using it in an idealized way, mainly as a tool for better structuring the objects of a given membrane system.

Membrane creation can be introduced both for systems with **Symbol objects** (6-9) and for systems with **String-objects** (10). We considered the case of symbol objects.

The P system with Active Membranes was investigated in the 2003 Visiting Faculty Research Program (VFRP) research Project (11). The model was enhanced in the 2004 VFRP research project and the algorithms for the Satisfiability problem (SAT) and the HPP problems are given in the Appendix. In this project the Membrane Creation Techniques were applied to develop the P System model. The following sections describe the model and its application.

The Model

A P system with Membrane Creation of degree (m, n) , $n \geq m \geq 1$, is a construct:

$$\Pi = (V, T, C, \mu, w_0, w_1, \dots, w_{(m-1)}, R_0, R_1, \dots, R_{(n-1)}),$$

Where:

1. V is an alphabet; it consists of both productive and non-productive objects;
2. $T \subseteq V$, is the output alphabet;
3. $C \cap V \neq \emptyset$, is the set of catalysts;
4. μ is a membrane structure consisting of m membranes, with the membranes and the regions labeled in a one- to-one manner with elements in a given set; here the labels are used as 0 (for the skin membrane), 1, ..., $(m-1)$;
5. w_i , $0 \leq i \leq (m-1)$, are multi-sets of objects over V associated with the regions 0, 1, ..., $(m-1)$ of μ ;
6. R_i , $0 \leq i \leq (n-1)$, are finite set of evolution rules over V .

An evolution rule is of two types:

- (a) If a is a single non-productive object, then the evolution rule is in the form $a \rightarrow v$ or $c a \rightarrow c v$, where $c \in C$, $a \in (V - C)$, $v = v'$ or $v = v' \delta$ or $v = v' \tau$, where v' is a multi-set of objects over $((V-C) \times (\text{here, out})) \cup ((V-C) \times (\text{in}_j \mid 1 \leq j \leq (n-1)))$, and δ, τ are special symbols not in V .

- (b) If a is a single productive object, then the evolution rule is in the form $a \rightarrow [{}_i v]_i$ or $c a \rightarrow c [{}_i v]_i$, where $c \in C$, $a \in (V - C)$, v is a multi-set of objects over $(V - C)$. The rule of the form $a \rightarrow [{}_i v]_i$ means that the object identified by a is transformed into the object identified by v , surrounded by a new membrane having the label i . No rule of the form $a \rightarrow [{}_0 v]_0$ can appear in any set R_i . During a computation the number of membranes can increase or decrease.

The membrane structure and the multi-sets in Π constitute the initial configuration of the system. One can pass from a configuration to another one by using the evolution rules. This is done in parallel: all objects, from all membranes, which can be the subject of local evolution rules, should evolve simultaneously. A rule can be used only if there are objects which are free at the moment one checks its applicability.

The application of a rule $c a \rightarrow c v$ in a region containing a multi-set w means to remove a copy of the object a in the presence of c (catalyst), provided that such copies exist, then follow the prescription given by v : If an object appears in v in the form (a, here) , then it remains in the same region; if it appears in the form (a, out) , then a copy of the object a will be introduced in the region of the membrane placed outside the region of the rule $c a \rightarrow c v$; if it appears in the form (a, in_i) , then a copy of a is introduced in the membrane with index i , if such a membrane exists inside the current membrane, otherwise the rule can not be applied. If the special symbol τ appears, then the thickness of the membrane which delimits the region in question is increased by 1. Initially, all membranes have the thickness 1. If a rule in a membrane of thickness 1 introduces the symbol τ , then the thickness of the membrane becomes 2. A membrane of thickness 2 does not become thicker by using further rules which introduce the symbol τ , but no object can enter or exit it. If a rule which introduces the symbol δ is used in a membrane (including the skin membrane) of thickness 1, then the membrane is dissolved. If the membrane had thickness 2, then it returns to thickness 1. When ever the skin membrane is dissolved, the whole membrane system will be destroyed. If at the same step one uses rules which introduce both δ and τ in the same membrane, then the membrane does not change its thickness. No object can be communicated through a membrane of thickness two, hence rules which introduce commands **out** and **in**, requesting such communications, can not be used. The communication has priority over changing the thickness. If at the same step an object should be communicated and a rule introduces the action τ , then the object is communicated and “after that” the membrane changes the thickness. When applying a rule $a \rightarrow [{}_i v]_i$ in a region j , a copy of a is removed and a membrane with the label i is created, containing the multi-set v , inside the region of membrane j . We can never create a skin membrane. Similarly, when applying a rule $c a \rightarrow c [{}_i v]_i$ in a region j , a copy of a , in presence of c (catalyst), is removed and a membrane with the label i is created, containing the multi set v , inside the region of membrane j .

A sequence of transitions between configurations of a given P system Π is called a **Computation** with respect to Π . A computation is **successful** if and only if it halts, that is, there is no rule applicable to the objects present in the last configuration. The result of

a successful computation is $\psi_T(w)$, is denoted by $Ps(\Pi)$ (from “Parikh set”) and we say that it is generated by Π . (Note that we take into account only the objects from T).

The model was applied to solve the following problem:

The Hamiltonian Path Problem (HPP) for Undirected Graphs.

Let $G = (U, E)$ be an undirected graph with n nodes ($n \geq 2$), where U is the set of nodes and E , the set of edges.

The Hamiltonian path problem determines whether or not there exists a Hamiltonian path in G , that is, whether or not there exists a path that passes through all the nodes in U exactly once. The HPP for undirected graphs is known to be Non Deterministic Polynomial, NP-Complete.

Let $U = (a_1, a_2, \dots, a_n)$. We now construct a P System with membrane creation of degree $(1, n + 1)$, where 1 is the initial number of membranes & $(n+1)$ is the total number of membranes used in the computation.

$$\Pi = (V, T, C, \mu, w_0, R_0, R_1, \dots, R_n),$$

Where $V = (a_i, a_i', f_i, f_i', d_i, d_i', c_i^j \mid 1 \leq i \leq n, 0 \leq j \leq n) \cup (Y) \cup$

$(t_i \mid 0 \leq i \leq 3n);$ (here a_i are productive objects).

$T = (Y);$

$C = \varphi;$

$\mu = [\quad]_0;$

$w_0 = (t_0, a_1, a_2, \dots, a_n);$

The set R_0 contains the following rules:

1. $t_i \rightarrow t_{i+1}, 0 \leq i \leq 3n - 1;$
2. $t_{3n} \rightarrow \delta;$
3. $c_i^j \rightarrow \lambda, 1 \leq j \leq n - 1, \forall_i;$
4. $c_i^n \rightarrow (Y, out), \forall_i;$
5. $a_i \rightarrow [\quad f_i', c_i^0, a_{j1}, \dots, a_{jk} \quad]_i, 1 \leq i \leq n;$

(a_i will create a new membrane with label i and transform into a multi-set of objects

$f_i', c_i^0, a_{j_1}, \dots, a_{j_k}$, where j_1, \dots, j_k are vertices adjacent to vertex i).

The set R_i , $1 \leq i \leq n$, contains the following rules:

$$1. a_j \rightarrow [{}_i f_j'', d_j, a_{j_1}', \dots, a_{j_k}']_j, 1 \leq j \leq n;$$

(a_j will create a new membrane with label j and transform into a multi-set of objects

$f_j', d_j', a_{j_1}', \dots, a_{j_k}'$, where j_1, \dots, j_k are vertices adjacent to vertex j).

$$2. d_i' \rightarrow d_i;$$

$$3. f_i' \rightarrow (f_i, in_{j_1}) (f_i, in_{j_2}) \dots (f_i, in_{j_k});$$

(Here j_1, \dots, j_k are vertices adjacent to vertex i).

$$4. f_j' \rightarrow (f_j, in_{j_1}) (f_j, in_{j_2}) \dots (f_j, in_{j_k}), \forall_j \neq i;$$

(Here j_1, \dots, j_k are vertices adjacent to vertex i).

$$5. a_j' \rightarrow a_j;$$

$$6. d_i' \rightarrow c_i^0;$$

$$7. f_i' \rightarrow \lambda \tau;$$

$$8. c_j^k \rightarrow (c_j^{k+1}, out), k \geq 0;$$

$$N_{(1, n+1)}(\Pi) = (Y^m, \text{ if the given graph has } m \text{ Hamiltonian paths.})$$

$$= \varphi \text{ otherwise.}$$

The system works as follows:

Initially the skin membrane contains all the productive objects a_1, a_2, \dots, a_n , where each productive object a_i , by using the rule $a_i \rightarrow [{}_i f_i', c_i^0, a_{j_1}, \dots, a_{j_k}]_i$, will create a membrane with index i and transform into the multi-set of objects

$$f_i', c_i^0, a_{j_1}, \dots, a_{j_k}.$$

In membrane i , a productive object a_j will create a membrane with index j and transform into a multi-set of objects $f_j'', d_j, a_{j_1}', \dots, a_{j_k}'$. In membrane i , f_j' will be replaced with f_j , d_j' will be replaced with d_j , and a_j with a_j . In the next step, d_j will be replaced with c_j^0 , a_j will create a membrane by using the rule $a_j \rightarrow [{}_j f_j'', d_j, a_{j_1}', \dots, a_{j_k}']_j$ and the process defined above will be repeated. When ever a new membrane is created, all f_j 's and f_i' 's, from its parent membrane, will move into that membrane. The role of f_j is to increase the thickness of the membrane j , so that no object will be sent out from that membrane. This will eliminate the effect of the path, which contains multiple copies of the same node, on

the result. For every rewriting step, c_j^k will be rewritten as c_j^{k+1} and sent out of the membrane i provided that the thickness of the membrane i is 1. In the skin membrane, $c_{j,i}^n$ (indicates that there exists a tour of path length n) will be replaced with Y and sent out. A counter t_i is used in the skin membrane, so that for every rewriting step it will be incremented. When ever the counter reaches t_{3n} , indicating the end of the computation, the counter will dissolve the whole membrane system by dissolving the skin membrane. If there exists a tour of length n , by traversing all nodes in the graph, then we get Y from the system.

Time Complexity: This algorithm takes $3n+1$ steps for generating the output.

The Algorithm: Hamiltonian Path Problem for Undirected Graphs.

Let us consider the graph $G = (U, E)$, with $U = \{a_1, a_2, a_3\}$ as shown Fig 1 below:

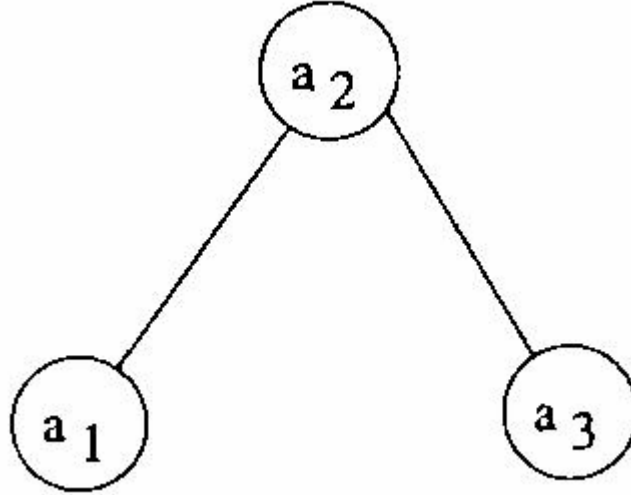


Figure 1: The Undirected Graph (HPP)

We now construct a P system with membrane creation of degree $(1, 4)$ as:

$$\Pi = (V, T, C, \mu, w_0, R_0, \dots, R_3),$$

Where $V = (a_i, a_i', f_i, f_i', d_i, d_i', c_i^j \mid 1 \leq i \leq 3, 0 \leq j \leq 3) \cup (t_i \mid 0 \leq i \leq 9) \cup (Y);$

$T = (Y);$

$C = \varnothing;$

$\mu = \begin{bmatrix} 0 & \end{bmatrix}_0;$

$$w_0 = (t_0, a_1, \dots, a_3);$$

$$\text{and } R_i, 0 \leq i \leq 3,$$

$$1. \left[{}_0 t_0 a_1 a_2 a_3 \right]_0$$

$$2. \left[{}_0 t_1 \left[{}_1 f_1' c_1^0 a_2 \right]_1 \left[{}_2 f_2' c_2^0 a_1 a_3 \right]_2 \left[{}_3 f_3' c_3^0 a_2 \right]_3 \right]_0$$

$$3. \left[{}_0 t_2 c_1^1 c_2^1 c_3^1 \left[{}_1 f_1' \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_1 \left[{}_2 f_2' \left[{}_1 f_1' d_1' a_2' \right]_1 \left[{}_3 f_3' d_3' a_2' \right]_3 \right]_2 \right. \\ \left. \left[{}_3 f_3' \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_3 \right]_0$$

$$4. \left[{}_0 t_3 \left[{}_1 \left[{}_2 f_1 f_2' d_2 a_1 a_3 \right]_2 \right]_1 \left[{}_2 \left[{}_1 f_1' f_2 d_1 a_2 \right]_1 \left[{}_3 f_2 f_3' d_3 a_2 \right]_3 \right]_2 \right. \\ \left. \left[{}_3 \left[{}_2 f_2' f_3 d_2 a_1 a_3 \right]_2 \right]_3 \right]_0$$

$$5. \left[{}_0 t_4 \left[{}_1 \left[{}_2 f_1 f_2' c_2^0 \left[{}_1 f_1' d_1' a_2' \right]_1 \left[{}_3 f_3' d_3' a_2' \right]_3 \right]_2 \right]_1 \right. \\ \left[{}_2 \left[{}_1 f_1' f_2 c_1^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_1 \left[{}_3 f_2 f_3' c_3^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_3 \right]_2 \\ \left. \left[{}_3 \left[{}_2 f_2' f_3 c_2^0 \left[{}_1 f_1' d_1' a_2' \right]_1 \left[{}_3 f_3' d_3' a_2' \right]_3 \right]_2 \right]_3 \right]_0$$

$$6. \left[{}_0 t_5 \left[{}_1 c_2^1 \left[{}_2 \left[{}_1 f_1 f_2 f_1' d_1 a_2 \right]_1 \left[{}_3 f_1 f_2 f_3' d_3 a_2 \right]_3 \right]_2 \right]_1 \right. \\ \left[{}_2 c_1^1 c_3^1 \left[{}_1 \left[{}_2 f_1 f_2 f_2' d_2 a_1 a_3 \right]_2 \right]_1 \left[{}_3 \left[{}_2 f_2 f_3 f_2' d_2 a_1 a_3 \right]_2 \right]_3 \right]_2 \\ \left. \left[{}_3 c_2^1 \left[{}_2 \left[{}_1 f_2 f_3 f_1' d_1 a_2 \right]_1 \left[{}_3 f_2 f_3 f_3' d_3 a_2 \right]_3 \right]_2 \right]_3 \right]_0$$

$$7. \left[{}_0 t_6 c_1^2 c_2^2 c_3^2 \left[{}_1 \left[{}_2 \left({}_1 f_2 f_1' c_1^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right)_1 \right. \right. \right. \\ \left. \left[{}_3 f_1 f_2 f_3' c_3^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_3 \right]_2 \right]_1 \\ \left[{}_2 \left[{}_1 \left({}_2 f_1 f_2' c_2^0 \left[{}_1 f_1' d_1' a_2' \right]_1 \left[{}_3 f_3' d_3' a_2' \right]_3 \right)_2 \right]_1 \left[{}_3 \left({}_2 f_3 f_2' c_2^0 \left[{}_1 f_1' d_1' a_2' \right]_1 \right. \right. \right. \\ \left. \left. \left[{}_3 f_3' d_3' a_2' \right]_3 \right)_2 \right]_3 \right]_2 \\ \left. \left[{}_3 \left[{}_2 \left[{}_1 f_2 f_3 f_1' c_1^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right]_1 \left({}_3 f_2 f_3' c_3^0 \left[{}_2 f_2' d_2' a_1' a_3' \right]_2 \right)_3 \right]_2 \right]_3 \right]_0$$

$$8. \left[{}_0 t_7 \left[{}_1 \left[{}_2 c_3^1 \left({}_1 c_1^0 \left[{}_2 f_2 f_1 f_2' d_2 a_1 a_3 \right]_2 \right)_1 \left[{}_3 \left[{}_2 f_1 f_2 f_3 f_2' d_2 a_1 a_3 \right]_2 \right]_3 \right]_2 \right]_1 \right.$$

$$\left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1 f_2 f_1' d_1 a_2 \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} f_1 f_2 f_3' d_3 a_2 \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_3 f_2 f_1' d_1 a_2 \right] \right. \right. \\ \left. \left. \left[\begin{matrix} 3 \\ 3 \end{matrix} f_3 f_2 f_3' d_3 a_2 \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2$$

$$\left[\begin{matrix} 3 \\ 2 \end{matrix} c_1^1 \left[\begin{matrix} 1 \\ 2 \end{matrix} f_2 f_3 f_1 f_2' d_2 a_1 a_3 \right] \right]_2 \left(\begin{matrix} 3 \\ 3 \end{matrix} c_3^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2 f_3 f_2' d_2 a_1 a_3 \right] \right)_3 \left[\begin{matrix} 3 \\ 2 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_0$$

$$9. \left[\begin{matrix} 0 \\ 2 \end{matrix} t_8 \left[\begin{matrix} 1 \\ 1 \end{matrix} c_3^2 \left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 1 \\ 1 \end{matrix} c_1^0 \left(\begin{matrix} 2 \\ 2 \end{matrix} f_1 f_2' c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1' d_1' a_2' \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} f_3' d_3' a_2' \right] \right)_2 \right) \right] \left[\begin{matrix} 3 \\ 2 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} f_1 f_3 f_2' c_2^0 \right. \right. \\ \left. \left. \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1' d_1' a_2' \right] \right)_1 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} f_3' d_3' a_2' \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 2 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left(\begin{matrix} 1 \\ 2 \end{matrix} f_2 f_1' c_1^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2' d_2' a_1' a_3' \right] \right)_1 \left[\begin{matrix} 3 \\ 3 \end{matrix} f_1 f_2 f_3' c_3^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2' d_2' a_1' a_3' \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 3 \\ 2 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_3 f_2 f_1' c_1^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2' d_2' a_1' a_3' \right] \right)_1 \left(\begin{matrix} 3 \\ 3 \end{matrix} f_2 f_3' c_3^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2' d_2' a_1' a_3' \right] \right)_3 \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 3 \\ 3 \end{matrix} c_1^2 \left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} f_3 f_2' c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1' d_1' a_2' \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} f_3' d_3' a_2' \right] \right)_2 \right] \left(\begin{matrix} 3 \\ 3 \end{matrix} c_3^0 \left(\begin{matrix} 2 \\ 2 \end{matrix} f_3 f_2' c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1' d_1' a_2' \right] \right. \right. \\ \left. \left. \left[\begin{matrix} 3 \\ 3 \end{matrix} f_3' d_3' a_2' \right] \right)_2 \right)_3 \right] \left[\begin{matrix} 3 \\ 2 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_0$$

$$10. \left[\begin{matrix} 0 \\ 2 \end{matrix} t_9 c_1^3 c_3^3 \left[\begin{matrix} 1 \\ 2 \end{matrix} \left(\begin{matrix} 1 \\ 1 \end{matrix} c_1^0 \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1 f_2 f_1' d_1 a_2 \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} f_1 f_2 f_3' d_3 a_2 \right] \right)_2 \right) \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_1 f_3 f_2 f_1' d_1 a_2 \right] \right. \right. \\ \left. \left. \left[\begin{matrix} 3 \\ 3 \end{matrix} f_1 f_3 f_2 f_3' d_3 a_2 \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 c_3^1 \left(\begin{matrix} 1 \\ 1 \end{matrix} c_1^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2 f_1 f_2' d_2 a_1 a_3 \right] \right)_1 \left[\begin{matrix} 3 \\ 2 \end{matrix} f_1 f_2 f_3 f_2' d_2 a_1 a_3 \right] \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 3 \\ 3 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 c_1^1 \left[\begin{matrix} 1 \\ 2 \end{matrix} f_3 f_2 f_1 f_2' d_2 a_1 a_3 \right] \right)_1 \left(\begin{matrix} 3 \\ 3 \end{matrix} c_3^0 \left[\begin{matrix} 2 \\ 2 \end{matrix} f_2 f_3 f_2' d_2 a_1 a_3 \right] \right)_3 \right)_2 \right] \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 1 \end{matrix} \right]_1 \\ \left[\begin{matrix} 3 \\ 3 \end{matrix} \left[\begin{matrix} 2 \\ 1 \end{matrix} \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_3 f_2 f_1' d_1 a_2 \right] \left[\begin{matrix} 3 \\ 1 \end{matrix} f_3 f_2 f_3' d_3 a_2 \right] \right)_2 \right] \left(\begin{matrix} 3 \\ 3 \end{matrix} c_3^0 \left(\begin{matrix} 2 \\ 2 \end{matrix} c_2^0 \left[\begin{matrix} 1 \\ 1 \end{matrix} f_3 f_2 f_1' d_1 a_2 \right] \right. \right. \\ \left. \left. \left[\begin{matrix} 3 \\ 3 \end{matrix} f_3 f_2 f_3' d_3 a_2 \right] \right)_2 \right)_3 \right] \left[\begin{matrix} 3 \\ 2 \end{matrix} \right]_2 \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]_0$$

11. Y^2

Y^2 indicates that two (2) Hamiltonian paths exist in the given graph.

The paths are: $a_1 - a_2 - a_3$ and $a_3 - a_2 - a_1$.

Note: Here a different notation () is used for membranes of thickness 2.

Conclusions

The mathematical model for the P system with Active Membranes was developed. The model was applied to solve the Hamiltonian Path Problem for Undirected Graphs and the relevant algorithms developed.

Recommendations for Follow on Research

The following Research Topics are identified for further investigation:

- 1) Development of a Simulation model in Digital Computer for P Systems with Active Membrane,
- 2) Design and develop experimental techniques to implement the model in Bio-Chemical Media.

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Appendix A: Algorithm to solve SAT problem

The computational steps for the construction of a P system for the propositional formula:

$$\beta = (x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee \sim x_2 \vee x_3) \wedge (x_1 \vee \sim x_2 \vee \sim x_3) \text{ with } n=3 \text{ and } m=3$$

are shown below:

Step 0:

The initial configuration of the system is given as:

$$[{}_4 [{}_3 [{}_2 [{}_1 [{}_0 c_0 a_1 a_2 a_3]_0^0]_1^0]_2^0]_3^0]_4^0$$

Step 1:

$$[{}_4 [{}_3 [{}_2 [{}_1 [{}_0 c_1 t_1 a_2 a_3]_0^+]_0^+ c_1 f_1 a_2 a_3]_0^-]_1^0]_2^0]_3^0]_4^0$$

The rules applied in parallel are:

$$[{}_0 c_0 \rightarrow c_1]_0^0 \text{ and}$$

$$\text{the division rule, } [{}_0 a_1]_0^0 \rightarrow [{}_0 t_1]_0^+ [{}_0 f_1]_0^-$$

Step 2:

Membrane 1 is divided due to the presence of the two copies of membrane 0 with opposite polarizations using the rule:

$$[{}_1 [{}_0]_0^+ [{}_0]_0^-]_1^0 \rightarrow [{}_1 [{}_0]_0^0]_1^+ [{}_1 [{}_0]_0^0]_1^-$$

The counter c_1 is replaced by c_2 . No new truth value is introduced at this step since membrane 0 is not of neutral polarity. We get,

$$[{}_4 [{}_3 [{}_2 [{}_1 [{}_0 c_2 t_1 a_2 a_3]_0^0]_1^+ [{}_1 [{}_0 c_2 f_1 a_2 a_3]_0^0]_1^-]_2^0]_3^0]_4^0$$

Step 3:

Membrane 0 is divided and the truth assignment of the variable a_2 also takes place according to the following rule:

$$[{}_0 a_2]_0^0 \rightarrow [{}_0 t_2]_0^+ [{}_0 f_2]_0^-$$

The counter c_2 is replaced by c_3 . Membrane 2 is also divided due to the presence of two membranes 1 with opposite polarity by the rule:

$$[{}_2 [{}_1]_1^+ [{}_1]_1^-]_2^0 \rightarrow [{}_2 [{}_1]_1^0]_2^+ [{}_2 [{}_1]_1^0]_2^-$$

We get:

$$\begin{aligned} & [{}_4 [{}_3 [{}_2 [{}_1 [{}_0 c_3 t_1 t_2 a_3]_0^+ [{}_0 c_3 t_1 f_2 a_3]_0^-]_1^0]_2^+ \\ & [{}_2 [{}_1 [{}_0 c_3 f_1 t_2 a_3]_0^+ [{}_0 c_3 f_1 f_2 a_3]_0^-]_1^0]_2^-]_3^0]_4^0 \end{aligned}$$

Step4:

Membrane 1 is divided since it contains two membranes 0 of opposite polarity using the rule:

$$[{}_1 [{}_0]_0^+ [{}_0]_0^-]_1^0 \rightarrow [{}_1 [{}_0]_0^0]_1^+ [{}_1 [{}_0]_0^0]_1^-$$

The counter c_3 is replaced by c_4 . Membrane 3 would also be divided since it contains membranes 2 of opposite polarity using the rule:

$$[{}_m [{}_{m-1}]_{m-1}^+ [{}_{m-1}]_{m-1}^-]_m^0 \rightarrow [{}_m [{}_{m-1}]_{m-1}^0]_m^+ [{}_m [{}_{m-1}]_{m-1}^0]_m^-$$

since here $m=3$. Thus, we get,

$$\begin{aligned} & [{}_4 [{}_3 [{}_2 [{}_1 [{}_0 c_4 t_1 t_2 a_3]_0^0]_1^+ [{}_1 [{}_0 c_4 t_1 f_2 a_3]_0^0]_1^-]_2^0]_3^0 \\ & [{}_3 [{}_2 [{}_1 [{}_0 c_4 f_1 t_2 a_3]_0^0]_1^+ [{}_1 [{}_0 c_4 f_1 f_2 a_3]_0^0]_1^-]_2^0]_3^0]_4^0 \end{aligned}$$

Step 5:

Membrane 0 is divided again and there occurs truth assignment of the variable a_3 . Also, membrane 2 is divided due to the presence of two opposite polarity membrane 1. Counter c_4 is replaced by c_5 . Thus, we get:

$$\begin{aligned}
& \left[\begin{array}{c} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_5 \\ t_1 \\ t_2 \\ t_3 \end{array} \right]_0^+ \left[\begin{array}{c} c_5 \\ t_1 \\ t_2 \\ f_3 \end{array} \right]_0^- \right]_1^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_5 \\ t_1 \\ f_2 \\ t_3 \end{array} \right]_0^+ \left[\begin{array}{c} c_5 \\ t_1 \\ f_2 \\ f_3 \end{array} \right]_0^- \right]_2^0 \left[\begin{array}{c} 1 \\ 0 \end{array} \right]_3^0 \\
& \left[\begin{array}{c} 3 \\ 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_5 \\ f_1 \\ t_2 \\ t_3 \end{array} \right]_0^+ \left[\begin{array}{c} c_5 \\ f_1 \\ t_2 \\ f_3 \end{array} \right]_0^- \right]_1^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_5 \\ f_1 \\ f_2 \\ t_3 \end{array} \right]_0^+ \left[\begin{array}{c} c_5 \\ f_1 \\ f_2 \\ f_3 \end{array} \right]_0^- \right]_2^0 \left[\begin{array}{c} 1 \\ 0 \end{array} \right]_3^0 \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_4^0
\end{aligned}$$

Step 6:

Membrane 1 is divided by the rule

$$\left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_1^+ \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_1^- \right]_1^0 \rightarrow \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_1^+ \right]_1^+ \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_1^- \right]_1^-$$

Membrane 3 is divided by the rule

$$\left[\begin{array}{c} m \\ m-1 \end{array} \left[\begin{array}{c} m-1 \\ m-1 \end{array} \right]_{m-1}^+ \left[\begin{array}{c} m-1 \\ m-1 \end{array} \right]_{m-1}^- \right]_m^0 \rightarrow \left[\begin{array}{c} m \\ m-1 \end{array} \left[\begin{array}{c} 0 \\ m-1 \end{array} \right]_{m-1}^0 \right]_m^0 \left[\begin{array}{c} m \\ m-1 \end{array} \left[\begin{array}{c} 0 \\ m-1 \end{array} \right]_{m-1}^0 \right]_m^0$$

since here $m=3$. The counter c_5 is replaced by c_6 . Thus we get,

$$\begin{aligned}
& \left[\begin{array}{c} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ t_1 \\ t_2 \\ t_3 \end{array} \right]_0^0 \right]_1^+ \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ t_1 \\ t_2 \\ f_3 \end{array} \right]_0^0 \right]_1^- \right]_2^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ t_1 \\ f_2 \\ t_3 \end{array} \right]_0^0 \right]_1^+ \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ t_1 \\ f_2 \\ f_3 \end{array} \right]_0^0 \right]_1^- \right]_2^0 \left[\begin{array}{c} 1 \\ 0 \end{array} \right]_3^0 \\
& \left[\begin{array}{c} 3 \\ 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ f_1 \\ t_2 \\ t_3 \end{array} \right]_0^0 \right]_1^+ \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ f_1 \\ t_2 \\ f_3 \end{array} \right]_0^0 \right]_1^- \right]_2^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ f_1 \\ f_2 \\ t_3 \end{array} \right]_0^0 \right]_1^+ \left[\begin{array}{c} 1 \\ 0 \end{array} \left[\begin{array}{c} c_6 \\ f_1 \\ f_2 \\ f_3 \end{array} \right]_0^0 \right]_1^- \right]_2^0 \left[\begin{array}{c} 1 \\ 0 \end{array} \right]_3^0 \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_4^0
\end{aligned}$$

Step 7:

No more truth assignments are possible, so membrane 0 cannot divide any further. The counter c_6 is replaced by c_7 . Membrane 2 is divided and we get,

$$\begin{aligned}
& \left[\begin{array}{c} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ t_1 \\ t_2 \\ t_3 \end{array} \right]_0^0 \right]_1^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ t_1 \\ t_2 \\ f_3 \end{array} \right]_0^0 \right]_1^0 \right]_2^+ \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ t_1 \\ f_2 \\ t_3 \end{array} \right]_0^0 \right]_2^+ \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ t_1 \\ f_2 \\ f_3 \end{array} \right]_0^0 \right]_2^+ \right]_2^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ f_1 \\ t_2 \\ t_3 \end{array} \right]_0^0 \right]_2^+ \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ f_1 \\ t_2 \\ f_3 \end{array} \right]_0^0 \right]_2^+ \right]_2^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ f_1 \\ f_2 \\ t_3 \end{array} \right]_0^0 \right]_2^+ \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \left[\begin{array}{c} c_7 \\ f_1 \\ f_2 \\ f_3 \end{array} \right]_0^0 \right]_2^+ \right]_2^0 \left[\begin{array}{c} 2 \\ 1 \\ 0 \end{array} \right]_3^0 \left[\begin{array}{c} 0 \\ 0 \end{array} \right]_4^0
\end{aligned}$$

Step8:

Membrane 3 is divided by the rule,

$$\left[\begin{matrix} m \\ m-1 \end{matrix} \right]_{m-1}^+ \left[\begin{matrix} m-1 \\ m-1 \end{matrix} \right]_{m-1}^- \left[\begin{matrix} 0 \\ m \end{matrix} \right] \rightarrow \left[\begin{matrix} m \\ m-1 \end{matrix} \right]_{m-1}^0 \left[\begin{matrix} 0 \\ m \end{matrix} \right] \left[\begin{matrix} m \\ m-1 \end{matrix} \right]_{m-1}^0 \left[\begin{matrix} 0 \\ m \end{matrix} \right]$$

The counter c_7 is replaced by c_8 . We get,

$$\begin{aligned} & \left[\begin{matrix} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ t_1 \\ t_2 \\ t_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ t_1 \\ t_2 \\ f_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ t_1 \\ f_2 \\ t_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ t_1 \\ f_2 \\ f_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ f_1 \\ t_2 \\ t_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ f_1 \\ t_2 \\ f_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ f_1 \\ f_2 \\ t_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \right] \left[\begin{matrix} c_8 \\ f_1 \\ f_2 \\ f_3 \end{matrix} \right]_{0}^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 0 \\ 3 \\ 4 \end{matrix} \right] \end{aligned}$$

Step 9:

The counter has reached its maximum limit since $2n+m-1 = 8$ and hence, the rule applied would be $\left[\begin{matrix} 0 \\ c_{2n+m-1} \end{matrix} \right]_0^0 \rightarrow t$, that is $\left[\begin{matrix} 0 \\ c_8 \end{matrix} \right]_0^0 \rightarrow t$.

Thus, all membrane 0 are dissolved. We get,

$$\begin{aligned} & \left[\begin{matrix} 4 \\ 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ t_2 \\ t_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ t_2 \\ f_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ f_2 \\ t_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ f_2 \\ f_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ f_1 \\ t_2 \\ t_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ f_1 \\ t_2 \\ f_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \\ & \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ f_1 \\ f_2 \\ t_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \\ 1 \end{matrix} \right] \left[\begin{matrix} t \\ f_1 \\ f_2 \\ f_3 \end{matrix} \right]_{1}^0 \left[\begin{matrix} 0 \\ 2 \\ 3 \end{matrix} \right] \left[\begin{matrix} 0 \\ 3 \\ 4 \end{matrix} \right] \end{aligned}$$

Step 10:

Now, the rule

$$\left[\begin{matrix} j \\ t_i \end{matrix} \right]_j^0 \rightarrow t_i, \text{ if } x_i \text{ appears in clause } C_j, 1 \leq i \leq n, 1 \leq j \leq m, \text{ and}$$

$$\left[\begin{matrix} j \\ f_i \end{matrix} \right]_j^0 \rightarrow f_i, \text{ if } \sim x_i \text{ appears in clause } C_j, 1 \leq i \leq n, 1 \leq j \leq m$$

is applied. Clause 1 is satisfied by all except fourth, seventh and eighth. Thus, remaining membrane 1 are dissolved. We get,

$$\left[\begin{matrix} 4 \\ 3 \\ 2 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ t_2 \\ t_3 \end{matrix} \right]_{2}^0 \left[\begin{matrix} 0 \\ 3 \end{matrix} \right] \left[\begin{matrix} 3 \\ 2 \end{matrix} \right] \left[\begin{matrix} t \\ t_1 \\ t_2 \\ f_3 \end{matrix} \right]_{2}^0 \left[\begin{matrix} 0 \\ 3 \end{matrix} \right]$$

$$\begin{aligned} & \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \right]_2^0 \left]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ t \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \right]_1^0 \left]_2^0 \right]_3^0 \\ & \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \right]_2^0 \left]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \right]_2^0 \right]_3^0 \\ & \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ t \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \right]_1^0 \left]_2^0 \right]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ t \end{smallmatrix} \begin{smallmatrix} t \\ f \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \begin{smallmatrix} f \\ t \end{smallmatrix} \right]_1^0 \left]_2^0 \right]_3^0 \right]_4^0 \end{aligned}$$

Step 11:

Clause 2 is satisfied by third and fifth truth assignments, so the corresponding membrane 2 are dissolved and we get,

$$\begin{aligned} & \left[\begin{smallmatrix} 4 \\ 3 \end{smallmatrix} \left[\begin{smallmatrix} 2 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_1 \\ \mathfrak{t}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_3 \\ \mathfrak{t}_3 \end{smallmatrix} \right]_2^0 \right]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t} \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_1 \\ \mathfrak{t}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_3 \\ \mathfrak{f}_3 \end{smallmatrix} \right]_2^0 \right]_3^0 \\ & \left[\begin{smallmatrix} 3 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_1 \\ \mathfrak{f}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_3 \\ \mathfrak{t}_3 \end{smallmatrix} \right]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \left[\begin{smallmatrix} 1 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_1 \\ \mathfrak{f}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_3 \\ \mathfrak{f}_3 \end{smallmatrix} \right]_1^0 \right]_2^0 \right]_3^0 \\ & \left[\begin{smallmatrix} 3 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_1 \\ \mathfrak{t}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_3 \\ \mathfrak{t}_3 \end{smallmatrix} \right]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t} \\ \mathfrak{f}_1 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_2 \\ \mathfrak{t}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_3 \\ \mathfrak{f}_3 \end{smallmatrix} \right]_2^0 \right]_3^0 \\ & \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \left[\begin{smallmatrix} 1 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_1 \\ \mathfrak{f}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{t}_3 \\ \mathfrak{t}_3 \end{smallmatrix} \right]_1^0 \right]_2^0 \right]_3^0 \quad \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix} \left[\begin{smallmatrix} 1 \\ \mathfrak{t} \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_1 \\ \mathfrak{f}_2 \end{smallmatrix} \begin{smallmatrix} \mathfrak{f}_3 \\ \mathfrak{f}_3 \end{smallmatrix} \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \end{aligned}$$

Step 12:

Clause 3 is satisfied by only third truth assignment and thus, one copy of t is left free in the skin membrane, corresponding to the truth assignment which satisfy the formula.

$$\begin{aligned} & \left[\begin{smallmatrix} 4 & 3 & 2 & t & t_1 & t_2 & t_3 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} 3 & 2 & t & t_1 & t_2 & t_3 \end{smallmatrix} \right]_3^0 \left[\begin{smallmatrix} 3 & 2 & t & t_1 & t_2 & f_3 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} t & t_1 & f_2 & t_3 \end{smallmatrix} \right]_3^0 \left[\begin{smallmatrix} 3 & 2 & 1 & t & t_1 & f_2 & f_3 \end{smallmatrix} \right]_1^0 \left[\begin{smallmatrix} 2 & 1 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} 3 \end{smallmatrix} \right]_3^0 \\ & \left[\begin{smallmatrix} 3 & t & f_1 & t_2 & t_3 \end{smallmatrix} \right]_3^0 \left[\begin{smallmatrix} 3 & 2 & t & f_1 & t_2 & f_3 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} 3 & 2 & 1 & t & f_1 & f_2 & t_3 \end{smallmatrix} \right]_1^0 \left[\begin{smallmatrix} 2 & 1 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} 3 & 2 & 1 & t & f_1 & f_2 & f_3 \end{smallmatrix} \right]_3^0 \left[\begin{smallmatrix} 2 & 1 \end{smallmatrix} \right]_2^0 \left[\begin{smallmatrix} 3 \end{smallmatrix} \right]_3^0 \left[\begin{smallmatrix} 4 \end{smallmatrix} \right]_4^0 \end{aligned}$$

Step 13:

t will be sent out of the system by the rule,

$$\left[\begin{array}{c} m+1 \\ t \end{array} \right]_{m+1}^0 \rightarrow \left[\begin{array}{c} m+1 \\ m+1 \end{array} \right]_{m+1}^+ t$$

Thus the formula is satisfied. This is the last step of the computation because no further rule can be applied. The skin becomes positively charged.

Algorithm to solve Hamiltonian Path Problem

The computational steps for the construction of the P system for the Graph,

$$G = (\{1,2,3,4\}, \{\{1,2\}, \{2,3\}, \{3,4\}\})$$

Here $n = 4$, so we will have 6 membranes, labeled from 0 to 5.

Step 0:

The initial configuration is given as:

$$\left[\begin{matrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} c_0 f \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]$$

Step 1:

The counter c_0 is incremented using Rule 1. Membrane 0 is divided using Rule 3.

$$\left[\begin{matrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} c_1 a_1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]^+ \left[\begin{matrix} c_1 a_2 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^- \left[\begin{matrix} c_1 a_3 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^+ \left[\begin{matrix} c_1 a_4 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$

Step 2:

The counter c_1 is incremented to c_2 . Membrane 1 is divided according to Rule 9. No rules of type 4 are applied to a_i here since the membrane 0 is not neutral.

$$\left[\begin{matrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} c_2 a_1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]^+ \left[\begin{matrix} c_2 a_2 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$
$$\left[\begin{matrix} 1 \\ 0 \end{matrix} \left[\begin{matrix} c_2 a_3 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]^+ \left[\begin{matrix} c_2 a_4 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$

Step 3:

The counter is incremented to c_3 . Rules of type 4 are applied to a_i . Membrane 2 is divided by Rule 9.

$$\left[\begin{matrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} c_3.1.a_2 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]^+ \left[\begin{matrix} c_3.2.a_3 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^+ \left[\begin{matrix} c_3.2.a_1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$
$$\left[\begin{matrix} 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} c_3.3.a_4 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right] \right]^+ \left[\begin{matrix} c_3.3.a_2 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$
$$\left[\begin{matrix} c_3.4.a_3 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^+ \left[\begin{matrix} c_3.4.a_1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \right]^-$$

Step 4:

The counter c_3 is replaced by c_4 . Rules of type 4 are again applied to a_2 and a_3 in neutral membranes 0. For a_i , in non-neutral membranes 0, rule of type 4 cannot be applied; hence, membrane 1 divides instead. Membrane 3 is divided using the rule of type 9.

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_4 .1.2. a_3 \right]_0^+ \left[{}_0 c_4 .1.2. a_1 \right]_0^- \right]_1^0 \right]_2^0 \right]_3^+ \right. \right. \\
& \quad \left. \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_4 .2. a_3 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 c_4 .2. a_1 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^- \right. \right. \\
& \quad \left. \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_4 .3. a_4 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 c_4 .3. a_2 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^+ \right. \right. \\
& \quad \left. \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_4 .4.3. a_4 \right]_0^+ \left[{}_0 c_4 .4.3. a_2 \right]_0^- \right]_1^0 \right]_2^0 \right]_3^- \right]_4^0 \right]_5^0
\end{aligned}$$

Step 5:

The counter c_4 is incremented to c_5 . Membrane 4 is divided by the rule,

$$\left[{}_n \left[{}_{n-1} \left[{}_{n-1} \left[{}_{n-1} \right]_{n-1}^+ \left[{}_{n-1} \right]_{n-1}^- \dots \left[{}_{n-1} \left[{}_{n-1} \right]_{n-1}^+ \right]_{n-1}^0 \right]_n^0 \rightarrow \left[{}_n \left[{}_{n-1} \left[{}_{n-1} \right]_{n-1}^0 \right]_{n-1}^0 \left[{}_n \left[{}_{n-1} \left[{}_{n-1} \right]_{n-1}^0 \right]_{n-1}^0 \right]_n^0 \dots \left[{}_n \left[{}_{n-1} \left[{}_{n-1} \right]_{n-1}^0 \right]_{n-1}^0 \right]_n^0$$

where $n = 4$.

Membrane 1 is divided in the case where membrane 0 is non-neutral. Membrane 2 is divided if membrane 1 is non-neutral. Rules of type 4 are applied to that a_i which is there in neutral membranes 0.

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_5 .1.2. a_3 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 c_5 .1.2. a_1 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^0 \right]_4^0 \right. \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_5 .2.3. a_4 \right]_0^+ \left[{}_0 c_5 .2.3. a_2 \right]_0^- \right]_1^0 \right]_2^+ \right]_2 \left[{}_1 \left[{}_0 c_5 .2.1. \right. \right. \\
& \quad \left. \left. a_2 \right]_0^0 \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_5 .3.4. a_3 \right]_0^0 \right]_1^0 \right]_2^+ \right]_2 \left[{}_1 \left[{}_0 c_5 .3.2. a_3 \right]_0^+ \left[{}_0 c_5 .3.2. \right. \right. \\
& \quad \left. \left. a_1 \right]_0^- \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_5 .4.3. a_4 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 c_5 .4.3. a_2 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^0 \right]_4^0 \right]_5^0
\end{aligned}$$

Step 6:

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_6 .1.2.3. a_4 \right]_0^+ \left[{}_0 c_6 .1.2.3. a_2 \right]_0^- \right]_1^0 \right]_2^+ \right. \right. \\
& \quad \left. \left[{}_2 \left[{}_1 \left[{}_0 c_6 .1.2.1. a_2 \right]_0^0 \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 c_6 .2.3. a_4 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 c_6 .2.3. a_2 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^+ \right.
\end{aligned}$$

$$\begin{aligned} & \left[\begin{matrix} 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} \mathbf{c}_6 \text{ 3.2. } \mathbf{a}_3 \end{matrix} \right]_0^0 \right]_1^+ \left[\begin{matrix} 1 \\ 0 \end{matrix} \left[\begin{matrix} \mathbf{c}_6 \text{ 3.2. } \mathbf{a}_1 \end{matrix} \right]_0^0 \right]_1^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_2^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_3^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_4^0 \\ & \left[\begin{matrix} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} \mathbf{c}_6 \text{ 4.3.4. } \mathbf{a}_3 \end{matrix} \right]_0^0 \right]_1^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \right]_2^+ \\ & \left[\begin{matrix} 2 \\ 1 \\ 0 \end{matrix} \left[\begin{matrix} \mathbf{c}_6 \text{ 4.3.2. } \mathbf{a}_3 \end{matrix} \right]_0^+ \left[\begin{matrix} 0 \end{matrix} \left[\begin{matrix} \mathbf{c}_6 \text{ 4.3.2. } \mathbf{a}_1 \end{matrix} \right]_0^- \right]_0^0 \right]_1^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_2^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_3^- \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_4^0 \left[\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \right]_5^0 \end{aligned}$$

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.1.2.3.a}_4 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 \text{c7.1.2.3.a}_2 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^+ \right. \\
& \quad \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.1.2.1.2.a}_3 \right]_0^+ \left[{}_0 \text{c7.1.2.1.2.a}_1 \right]_0^- \right]_1^0 \right]_2^0 \right]_3^- \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.2.3.4.a}_3 \right]_0^0 \right]_1^0 \right]_2^+ \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.2.3.2.a}_3 \right]_0^+ \left[{}_0 \text{c7.2.3.2.a}_1 \right]_0^- \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.2.1.2.a}_3 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 \text{c7.2.1.2.a}_1 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.3.4.3.a}_4 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 \text{c7.3.4.3.a}_2 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.3.2.3.a}_4 \right]_0^+ \left[{}_0 \text{c7.3.2.3.a}_2 \right]_0^- \right]_1^0 \right]_2^+ \right. \right. \\
& \quad \left. \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.3.2.1.a}_2 \right]_0^0 \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \quad \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.4.3.4.3.a}_4 \right]_0^+ \left[{}_0 \text{c7.4.3.4.3.a}_2 \right]_0^- \right]_1^0 \right]_2^0 \right]_3^+ \right. \\
& \quad \left[{}_3 \left[{}_2 \left[{}_1 \left[{}_0 \text{c7.4.3.2.a}_3 \right]_0^0 \right]_1^+ \left[{}_1 \left[{}_0 \text{c6.4.3.2.a}_1 \right]_0^0 \right]_1^- \right]_2^0 \right]_3^- \right]_4^0
\end{aligned}$$

No more rules are possible for a_i in membrane 0, as c_i cannot increase further. Thus, Rule 2 is applied and the counter is transformed to t as membrane 0 dissolves.

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$$\begin{aligned}
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.4.3.a}_4 \right]_1^0 \right]_2^+ \left[{}_2 \left[{}_1 \text{t.3.4.3.a}_2 \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.2.3.a}_4.\text{t.3.2.3.a}_2 \right]_1^0 \right]_2^0 \right]_3^+ \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.2.1.a}_2 \right]_1^0 \right]_2^0 \right]_3^- \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.4.3.4.3.a}_4.\text{t.4.3.4.3.a}_2 \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.4.3.2.a}_3 \right]_1^0 \right]_2^+ \left[{}_2 \left[{}_1 \text{t.4.3.2.a}_1 \right]_1^0 \right]_2^- \right]_3^0 \right]_4^0 \right]_5^0
\end{aligned}$$

Step 9:

Rule 5 is applied and all the a_i 's are converted into i 's. Membrane 1 dissolves if 1 is present according to rule 6. Membrane 1 also dissolves if a_1 is present according to rule 7.

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \left[{}_2 \text{t.1.2.3.4} \right]_2^0 \right]_3^+ \left[{}_3 \left[{}_2 \text{t.1.2.3.2} \right]_2^0 \right]_3^- \right]_4^0 \right]_5^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \text{t.1.2.1.2.3.t.1.2.1.2.1} \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.2.3.4.3} \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \text{t.2.3.2.3.t.2.3.2.1} \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \text{t.2.1.2.3} \right]_2^0 \right]_3^+ \left[{}_3 \left[{}_2 \text{t.2.1.2.1} \right]_2^0 \right]_3^- \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.4.3.4} \right]_1^0 \right]_2^0 \right]_3^+ \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.4.3.2} \right]_1^0 \right]_2^0 \right]_3^- \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.3.2.3.4.t.3.2.3.2} \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \text{t.3.2.1.2} \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.4.3.4.3.4.t.4.3.4.3.2} \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.4.3.2.3} \right]_1^0 \right]_2^0 \right]_3^+ \left[{}_3 \left[{}_2 \text{t.4.3.2.1} \right]_2^0 \right]_3^- \right]_4^0 \right]_5^0
\end{aligned}$$

Step 10:

Membrane 2 dissolves if 2 is present.

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \left[{}_3 \text{t.1.2.3.4} \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \text{t.1.2.3.2} \right]_3^0 \right]_4^0 \right]_5^0 \\
& \left[{}_4 \left[{}_3 \text{t.1.2.1.2.3.t.1.2.1.2.1} \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{t.2.3.4.3} \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \text{t.2.3.2.3.t.2.3.2.1} \right]_3^0 \right]_4^0
\end{aligned}$$

$$\begin{aligned}
& \left[{}_4 \left[{}_3 \text{ t.2.1.2.3 } \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \text{ t.2.1.2.1 } \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.4 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.2.3.4.t.3.2.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \text{ t.3.2.1.2 } \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.4.3.4.t.4.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.2.3 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \text{ t.4.3.2.1 } \right]_3^0 \right]_4^0 \right]_5^0
\end{aligned}$$

Step 11:

Membrane 3 dissolves if 3 is present.

$$\begin{aligned}
& \left[{}_5 \left[{}_4 \text{ t.1.2.3.4 } \right]_4^0 \left[{}_4 \text{ t.1.2.3.2 } \right]_4^0 \left[{}_4 \text{ t.1.2.1.2.3.t.1.2.1.2.1 } \right]_4^0 \right. \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.2.3.4.3 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \text{ t.2.3.2.3.t.2.3.2.1 } \right]_4^0 \\
& \left[{}_4 \text{ t.2.1.2.3 } \right]_4^0 \left[{}_4 \left[{}_3 \text{ t.2.1.2.1 } \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.4 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.2.3.4.t.3.2.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \text{ t.3.2.1.2 } \right]_4^0 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.4.3.4.t.4.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left. \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.2.3 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \text{ t.4.3.2.1 } \right]_4^0 \right]_5^0
\end{aligned}$$

Step 12:

Membrane 4 dissolves if 4 is present.

$$\begin{aligned}
& \left[{}_5 \text{ t.1.2.3.4 } \left[{}_4 \text{ t.1.2.3.2 } \right]_4^0 \left[{}_4 \text{ t.1.2.1.2.3.t.1.2.1.2.1 } \right]_4^0 \right. \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.2.3.4.3 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \text{ t.2.3.2.3.t.2.3.2.1 } \right]_4^0 \\
& \left[{}_4 \text{ t.2.1.2.3 } \right]_4^0 \left[{}_4 \left[{}_3 \text{ t.2.1.2.1 } \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.4 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.3.2.3.4.t.3.2.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left[{}_4 \text{ t.3.2.1.2 } \right]_4^0 \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.4.3.4.t.4.3.4.3.2 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \\
& \left. \left[{}_4 \left[{}_3 \left[{}_2 \left[{}_1 \text{ t.4.3.2.3 } \right]_1^0 \right]_2^0 \right]_3^0 \right]_4^0 \text{ t.4.3.2.1 } \right]_5^0
\end{aligned}$$

Step 13:

There are two copies of t in the skin membrane. One of the copies of t goes out of the skin membrane. The computation stops, because there are no more rules applied. Thus, the Hamiltonian Path Problem is solved.